

# Formalizing Timing Diagram Requirements: in Discrete Duration Calculus

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# Formal Requirement Modelling and Analysis

- Domain: Behavioural requirements over Embedded system controllers/Digital Hardware modules
- **Formal specification** notation: some form of **temporal logic**
- **In practice**: Informal specification, Heterogenous, Visual (UML timing diagrams, state machines, MSC ), Structured text.

## Promise of Formal Specification

- **Unambiguous**
- **Requirement Analysis**: Consistency, Model Visualization, Completeness, Implication checking, Realizability checking.
- **Verification**: Model checking, Runtime verification, Automatic test suite generation.
- **Synthesis**: Automatic construction of controller which matches the specification.

In this paper: A logic for formalizing timing diagram requirements

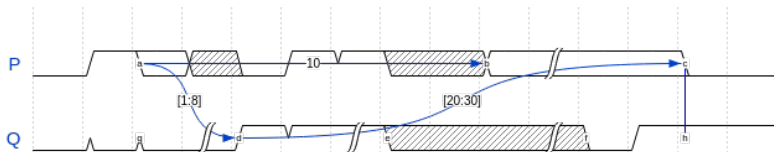
## Fisler 2005

The less than satisfactory adoption of formal methods in timing diagram domain can be partly attributed to the gulf that exists between graphical timing diagrams and textual temporal logic – expressing various timing dependencies that can exist among signals that can be illustrated so naturally in timing diagrams is rather tedious in temporal logics.

As a result, hardware designers use timing diagrams informally without any well defined semantics which make them unamenable to automatic design verification techniques.

- Timing diagrams
- From Timing diagrams to Temporal Logic
- SECENL  $\subset$  QDDC
- Encoding Timing diagrams in SECENL, and comparison
- Elementary complexity of SECENL automaton construction.  
Tool **DCTOOLS**.

# Timing Diagrams



## Components of Timing Diagrams

- Timing diagram  $T = (W, \Sigma, C, \Theta)$
- $\Sigma$  set of propositions
- $W(p)$  gives the **waveform** of  $p \in \Sigma$
- $\Theta$  a set of named positions in waveforms.
- $C$  set of constraints between pairs of named positions.
  - Precedence constraints
  - Timing constraints

waveform  $W_p$  - 01a : 2x011xb : x2|220c : 00

waveform  $W_q$  - 00a : 0|d : 11|e : xxx|f : 01c : 11

timing constraints:  $d-a \in [1:8]$ ,  $c-d \in [20:30]$ ,  $b-a \in [10:10]$

# Timing Diagram Textual Syntax

Timing diagram  $T = (W, \Sigma, C, \Theta)$

## Waveform syntax

$\pi := 0 \mid 1 \mid 2 \mid 0| \mid 1| \mid 2| \mid x| \mid u : \pi \mid \pi_1\pi_2,$   
where  $u \in \Theta$

0 denotes *low*, 2 *any*, and “|” the *stuttering* operator. Thus, 1| gives arbitrarily long high signal, and x|, arbitrarily long unchange.

## Constraints

- $C$  is a list of constraints
- Example constraint:  $(c, d, [20 : 30])$
- Set of Constraints  $\Theta \times \Theta \times \text{Intv}(\mathbb{N})$
- Syntax is adapted from [Wavedrom2](#) which draws the picture.
- **DCTOOLS** translates TD to logic SECENL.

# Timing Diagram Logics

- **LTL** [Dill, Emerson,1997] and **CTL**
- Extensions: **Timing Diagram Logic** [Fisler,1999], **Pipeline operator** [Chockler, Fisler,2005], **SRTD** [Amla,Emerson,Kurshan,Namjoshi,2000]
- **PSL/Sugar (IEEE1850 [2005])**: formulated initially by Accelera Consortium
- **Interval Temporal Logic** [Moszkowski, 1983]  
**Duration Calculus** [Zhou, Hoare, Ravn, 1990]
- In this paper, **SECENL** a subset of **QDDC** [P.,1996] and **CTL\*(DC)** [P., 2001]

# Discrete-time Duration Calculus

QDDC logic of finite (non-empty) state sequences.

req	1	0	1	1	0
ack	0	0	0	0	1

We define  $\sigma, [b, e] \models D$ .

**Example**  $\langle \text{req} \rangle \wedge [\text{!ack}] \wedge \langle \text{ack} \rangle$

- Interval temporal logic
- Quantitative Measurements of Time

**Example** In any interval of 20 or more cycles where request is continuously high there must be at least 3 *ack* signals.

$$[] ( [] [\text{req}] \ \&\& \ \text{slen} \geq 20 \Rightarrow \text{scount ack} \geq 3 )$$



Let  $P \in Prop(\Sigma)$ ,  $c \in \mathbb{N}$ ,  $D1, D2 \in QDDC$ . Let  $\in \{ \leq, <, =, >, \geq \}$  Then syntax of QDDC:

$\langle P \rangle \mid [[P]] \mid \text{slen } \sim c \mid \text{scount } P \sim c \mid$   
 $D1 \wedge D2 \mid D^* \mid D1 \ \&\& \ D2 \mid !D \mid$   
(exists P. D)

# QDDC: Syntax and Semantics

$\sigma, [b, e] \models \langle P \rangle$  iff  $b = e$  and  $\sigma, b \models P$

$\sigma, [b, e] \models [[P]]$  iff for all  $t : b \leq t \leq e$ .  $\sigma, t \models P$

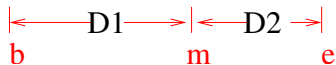
$\sigma, [b, e] \models [P]$  iff  $b < e$  and for all  $t : b \leq t < e$ .  $\sigma, t \models P$

$\sigma, [b, e] \models D1 \wedge D2$  iff for some  $m : b \leq m \leq e$ .

$\sigma, [b, m] \models D1$  and  $\sigma, [m, e] \models D2$



If for some  $m$



**Example:**  $[P] \wedge [!P] \wedge [[P]]$

A valid formula:  $\langle P \rangle \iff \langle P \rangle \wedge \langle P \rangle \wedge \langle P \rangle$

# Syntax and Semantics (2)

## Derived Operators

- For some subinterval D:  $\langle \rangle D \stackrel{\text{def}}{=} \text{true} \wedge D \wedge \text{true}$
- For all subintervals D:  $[] D \stackrel{\text{def}}{=} ! \langle \rangle ! D$

Validity in execution  $\sigma \models D$  iff  $\sigma, [0, \#\sigma - 1] \models D$

Example:  $[] (\langle \text{down}(P) \rangle \wedge [!P] \wedge \langle \text{up}(P) \rangle) \Rightarrow ! \langle \rangle ([!R] \wedge [R])$



Measurement Terms `slen` | `scount P`

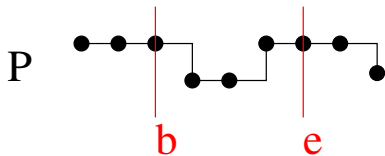
$$\text{eval}(\text{slen})(\sigma, [b, e]) \stackrel{\text{def}}{=} e - b$$

$$\text{eval}(\text{scount } P)(\sigma, [b, e]) \stackrel{\text{def}}{=} \sum_{i=b}^e \left\{ \begin{array}{l} 1 \text{ if } \sigma, i \models P \\ 0 \text{ otherwise} \end{array} \right\}$$

Measurement Formula `mt op c`

where  $op \in \{ < \mid > \mid = \mid \leq \mid \geq \}$ .

# Measuring Counts and Durations



$$\text{eval}(\text{slen}) = 4$$

$$\text{eval}(\text{scount } P) = 3$$

## Examples

```
[] ( [[req]] && slen >= 20 => scount ack >= 3)
```

Between any two  $P$  phases there are at least 300 cycles.

```
[] ( < down(P) > ^ [!P] ^ < up(P) > => (slen >= 300) )
```

- Minimum Separation
- Upper bound
- Persistence
- Arrow operators [Ravn94]

Quantification **exists**  $p: D$

$\sigma, [b, e] \models (\text{exists } p: D) \text{ iff } \sigma', [b, e] \models D$  for some  $p$ -variant  $\sigma'$

## Theorem (Automata Theoretic Decidability of QDDC)

- For each  $D \in \text{QDDC}$  we can effectively construct finite state automaton  $A_D$  such that  $L(D) = L(A_D)$ .
- For each FSM  $A$  we can effectively construct  $D_A \in \text{QDDC}$  such that  $L(A) = L(D_A)$ .

Tool DCVALID – next slide.

## Problem

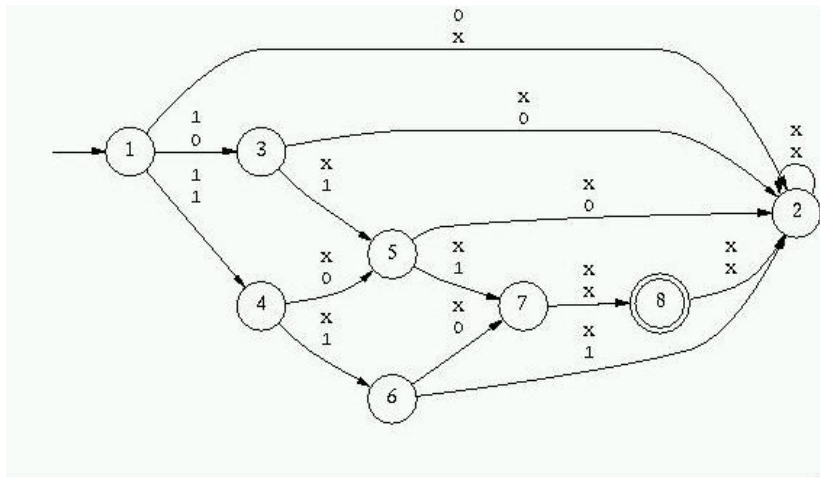
Size of minimum automaton can be non-elementary in size of formula in the worst case. Thus formula of size  $n$  can give minimal automaton of size  $O(2^{2^{\dots}})$ , tower of height  $n$ .

- Constructs deterministic finite state automaton  $\mathcal{A}(D)$  for QDDC formula  $D$ .
- The automaton is used as a synchronous observer to model check QDDC properties of Esterel, SMV, Verilog, SCADE/Lustre and SAL models.
- Uses efficient BDD-based representation of automata using MONA.
- Constructs automaton for formula bottom up keeping each automaton in minimal deterministic form.

[RTTOOLS2001, TACAS2001, SLAP2002, AVOCS2004, FSTTCS2005, TACAS2006]

# DCVALID Example

$$(\langle P \rangle \curvearrowright true) \wedge (slen = 4) \wedge (sdurQ = 2)$$

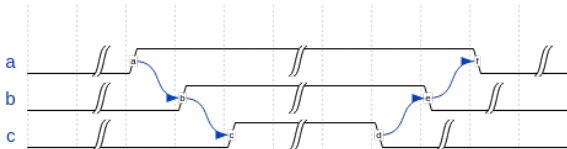




# SECE: Semi Extended Chop Expressions

- SECE is QDDC without **negation** and **quantification** operators.

$\langle P \rangle \mid [[P]] \mid \text{slen op } c \mid \text{scount } P \text{ op } c \mid$   
 $D1 \wedge D2 \mid D^* \mid D1 \ \&\& \ D2$



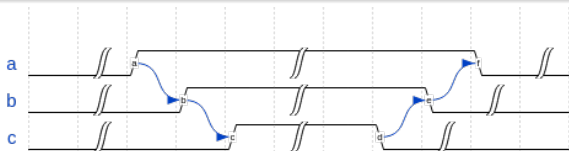
- SECE adequately captures a collection of waveforms (i.e. W)

$[!a] \wedge ([a]) \wedge [[!a]] \ \&\& \ [!b] \wedge ([b]) \wedge [[!b]] \ \&\&$   
 $[!c] \wedge ([c]) \wedge [[!c]]$

# Nominals: Capturing Waveform Constraints

- Nominals are propositions which uniquely mark specific positions in word.
- Used for synchronization between formulae.
- Let  $D$  be a SECE formula over  $\Sigma \cup \Theta$ .
- $(\text{ex1 } u: D)$  where  $D \in \text{SECE}$  and  $u \in \Theta$  is called SECEN formulas.
- $(\text{ex1 } u: D) = (\text{exists } u: \text{scount}(u)=1 \ \&\& \ D)$   
 $(\text{all1 } u: D) = (\text{all } u: (\text{scount}(u)=1 \Rightarrow D))$

# Timing Diagrams to SECEN



## SECEN Formula

```
ex1 ua,ub,uc,va,vb,vc:
```

```
-- waveforms
```

```
  [!a] ^<ua> ^ [a] ^<va> ^ [[!a]]  &&
```

```
  [!b] ^<ub> ^ [b] ^<vb> ^ [[!b]]  &&
```

```
  [!c] ^<uc> ^ [c] ^<vc> ^ [[!c]]  &&
```

```
-- constraints
```

```
  true ^<ua> ^ slen>0 ^<ub> ^ true  &&
```

```
  true ^<ub> ^ slen>0 ^<uc> ^ true  &&
```

```
  true ^<vc> ^ slen>0 ^<vb> ^ true  &&
```

```
  true ^<vb> ^ slen>0 ^<va> ^ true
```

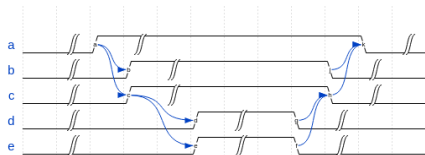
Size  $O(n)$

# Equivalent Formula without Nominals

$$\begin{aligned} & [!a \ \&\& \ !b \ \&\& \ !c] \ \wedge \ [a \ \&\& \ !b \ \&\& \ !c] \ \wedge \ [a \ \&\& \ b \ \&\& \ !c] \ \wedge \\ & [a \ \&\& \ b \ \&\& \ c] \ \wedge \\ & [!a \ \&\& \ !b \ \&\& \ !c] \ \wedge \ [a \ \&\& \ !b \ \&\& \ !c] \ \wedge \ [a \ \&\& \ b \ \&\& \ !c] \end{aligned}$$

Size  $O(n^2)$ .

## Example 2



### SECEN Formula

```
ex1 ua,ub,uc,va,vb,vc:
```

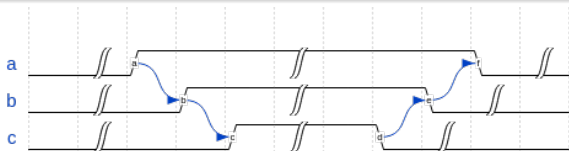
```
  (![a] ^ <ua> ^ [a] ^ <va> ^ [[!a]]) &&  
  (![b] ^ <ub> ^ [b] ^ <vb> ^ [[!b]]) && ...
```

```
-- constraints
```

```
  true ^ <ua> ^ slen > 0 ^ <ub> ^ true &&   true ^ <ua> ^ slen > 0 ^ <uc> ^ true &&  
  true ^ <uc> ^ slen > 0 ^ <ud> ^ true &&   true ^ <uc> ^ slen > 0 ^ <ue> ^ true &&  
  true ^ <vd> ^ slen > 0 ^ <vc> ^ true &&   true ^ <ve> ^ slen > 0 ^ <vc> ^ true &&  
  true ^ <vc> ^ slen > 0 ^ <va> ^ true &&   true ^ <vb> ^ slen > 0 ^ <va> ^ true
```

Writing this without nominals is tricky!

# Unordered Stack



## SECEN Formula

```
ex1 ua,ub,uc,va,vb,vc:
```

```
  [!a] ^<ua> ^ [a] ^<va> ^ [[!a]]  &&  ..
```

```
  [!c] ^<uc> ^ [c] ^<vc> ^ [[!c]]  &&
```

```
-- constraints
```

```
  ext ^<u1> ^ ext ^<u2> ^ ext ^<u3> ^ ext ^  
    <v3> ^ ext ^<v2> ^ ext ^<v1> ^ ext  &&
```

```
  Bijection(ua,ub,uc,va,vb,vc,u1,u2,u3,v1,v2,v3)
```

Size  $O(n^2)$

Stating this without nominals requires disjunction over all possible stack orders. Size  $O(n!)$

## Main Features

- Natural and **compositional** translation of timing diagrams into *SECEN*
- Exponential succinctness as compared with *SECE* (and *SERE* of PSL)
- Elementary automaton construction.

## Theorem

*For every  $D \in \text{SECEN}$  of size  $n$ , we can construct  $A(D)$  of size  $2^{2^{2^n}}$  such that  $L(D) = L(A(D))$ .*

# SECENL: Specifying Limited Liveness

- Modalities of occurrence of patterns of behaviour  
Inspired by LSC of UML 2.0.
- Making good things happen – within known bounds.

## Syntax of SECENL formula $\phi$

Let  $D_i$  be SECEN formulas. Let  $\Theta$  be a set of nominals.

**pref**( $D$ )

**anti**( $D$ )

**init** $\forall^1\Theta : (D_1/D_2)$

**implies** $\forall^1\Theta : (D_1 \rightsquigarrow D_2)$

**follows** $\forall^1\Theta : (D_1 \rightsquigarrow D_2/D_3)$

**triggers** $\forall^1\Theta : (D_1 \rightsquigarrow D_2/D_3)$

$\phi_1 \wedge \phi_2 \mid \neg\phi$

A SECENL formula  $\phi$  is equivalent to a QDDC formula (using negations) called  $\aleph(\phi)$ .



- **anti**( $D$ ) – pattern  $D$  must not occur anywhere in behavior.

$$\mathbb{N}(\mathbf{anti}(D)) = \square (!D)$$

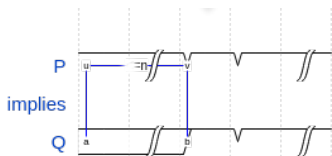
- **implies** $\forall^1\Theta : (D_1 \rightsquigarrow D_2)$

$$\mathbb{N}(\mathbf{implies}\forall^1\Theta : (D_1 \rightsquigarrow D_2)) = \\ \square (\text{all1 } \Theta : (D_1 \Rightarrow D_2))$$

- **follows** $\forall^1\Theta : (D_1 \rightsquigarrow D_2/D_3)$

$$\mathbb{N}(\mathbf{follows}\forall^1\Theta : (D_1 \rightsquigarrow D_2/D_3)) = \\ \square (\text{all1 } \Theta, u : (D_1 \wedge u \wedge D_3 \Rightarrow \text{true} \wedge u \wedge D_2 \wedge \text{true}))$$

# Example: Lags(P,Q,n)



SECENL formula:

**implies** $\forall^1 u$  :

( ([P] && slen=n) ^<u>^ [[P]])  $\rightsquigarrow$   
true ^<u>^ [[Q]] )

Minepump Example:

```
lags(HCH4,Alarm,3) && lags(HH20,Alarm,3) &&  
lags(!HCH4 && !HH20),!ALARM,3)
```

# Other Properties

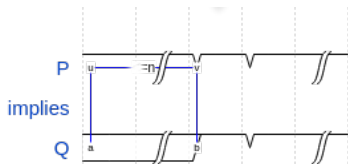


Figure : lags(P,Q,n).

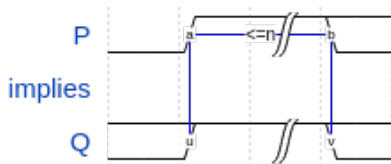


Figure : track(P,Q,n)

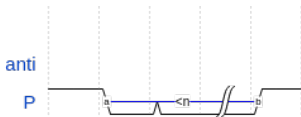


Figure : sepration(P,n)

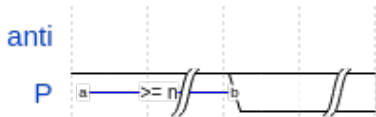


Figure : upperbound(P,n)

## Theorem (Elementary Automaton Construction)

*For every  $D \in \text{SECENL}$  of size  $n$ , we can construct  $A(D)$  of size  $2^{2^{2^{2^{2^n}}}}$  (tower of height 5) such that  $L(D) = L(A(D))$ .*

In practice not so bad! Compare this with PSL with SERE which gives tower of height 4.

# DCTOOLS: Architecture

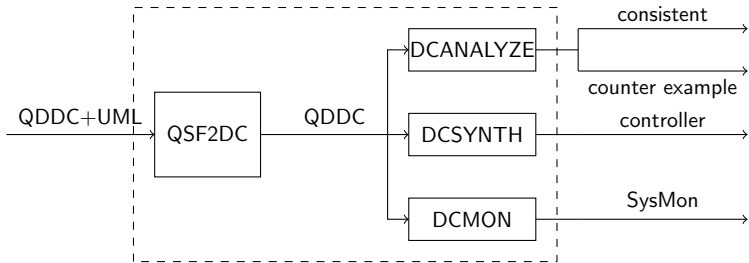
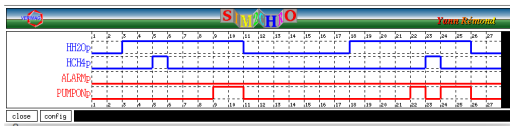


Figure : DCTOOLS.

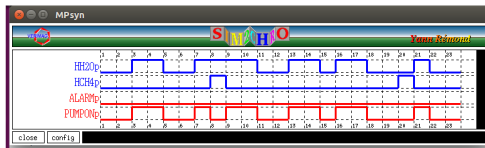
- Minepump Controller Specification (proceedings)  
Automatic Controller Synthesis
- Synchronous Bus Arbiter with diverse latency properties  
Automatic Controller Synthesis, Automatic Monitor Synthesis
- Alarm annunciation System for a plant
- Discordance logic for a plant
- Complete AMBA Bus AHB arbiter specification (in progress)
- Specification of self navigating and parking robot car controller  
Automatic controller synthesis (in progress)

# Synthesis with Soft Goals

Soft requirement:  $(!\$PUMPON\$) \gg (!\$ALARM\$)$



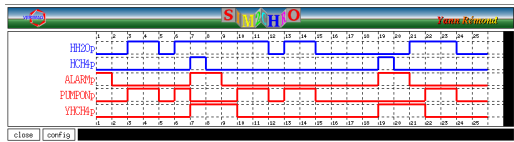
Soft Requirement:  $\$PUMPON\$ \gg (!\$ALARM\$)$



Soft Requirement:

$$((!YHCH4p\$) | (!PUMPONp\$)) \gg (\$PUMPONp\$)$$

where  $YHCH4p$  :  $\text{true}^{\wedge}(\text{slen}=2 \ \&\& \ \langle \rangle \langle HCH4p \rangle)$





# DCSynth Performance

Problem	Lily		Acacia+		DCSynth	
	time (Sec)	Memory /States	time (Sec)	Memory /States	time (Sec)	Memory /States
$Arb^{hard}(4, 4)$	161.9	172.6/ 108	0.4	29.8/ 55	0.09	5.0/ 50
$Arb^{hard}(5, 5)$	TO <sup>a</sup>	-	11.4	71.9/ 293	4.8	33.4/ 432
$Arb^{hard}(6, 6)$	-	-	TO	-	80	1053.0/ 4802
$Arb^{hard}(7, 7)$	-	-	TO	-	-	MO <sup>a</sup>
$Arb^{tok}(8)$	TO	-	46.44	77.9/ 73	1.9	12.8/ 8
$Arb^{tok}(10)$	TO	-	NC <sup>a</sup>	-	137	53.0/10
$Arb^{tok}(12)$	TO	-	NC	-	TO	255.0/ 12
MinePump	TO	-	NC	-	0.06	50/ 32

<sup>a</sup>TO=timeout, MO=memory out and NC=synthesis inconclusive

# Combining SECENL and LTL

- Let  $\phi \in \text{SECENL}$ . Then  $\text{LTL}[\text{SECENL}]$  contains LTL formulae where SECENL formulae are used as propositional letters.
- Semantics: Given finite or infinite behaviour  $\rho$  and  $i \in \text{dom}(\rho)$

$$\rho, i \models \phi \text{ iff } \rho, [0, i] \models \phi$$

- Easy to model check!
- Tool `ctldc` modelchecks  $\text{LTL}[\text{SECENL}]$  specification against LUSTRE or SMV model by using underlying model checker (see [P. TACAS2001]).
- All the standard LTL synthesis algorithms can be extended to  $\text{LTL}[\text{SECENL}]$ .

- A logic for requirements is a pragmatic choice between expressiveness and decision complexity.
- SECENL is a proposal which differs from state-of-the-art-logics like PSL or Regular LTL.
  - Semi Extended Chop Expressions with intersection and counting
  - Nominals for synchronization
  - Limited Liveness Properties: Interval logic connectives and implication without nesting.
- LTL[SECENL] is a simpler combination of LTL and SECENL. Easy to model check or synthesize.
- Tool support with DCTOOLS.