

# An in-Depth Investigation of Interval Temporal Logic Model Checking with Regular Expressions

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# Point-based vs. interval-based model checking

- Model checking is usually **point-based**:
  - properties express requirements over points (snapshots) of a computation (states of the state-transition system)
  - they are specified by means of point-based temporal logics such as LTL and CTL
- **Interval-based** model checking:
  - Interval-based properties express conditions on computation stretches: accomplishments, actions with duration, and temporal aggregations
  - they are specified by means of interval temporal logics, e.g., **HS**

# The logic HS

HS features a modality for any Allen ordering relation between pairs of intervals (except for equality)

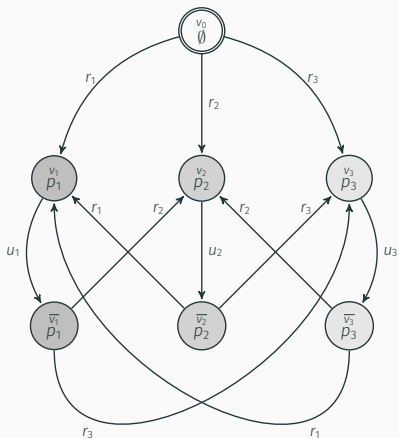
Allen rel.	HS	Definition	Example
<i>meets</i>	$\langle A \rangle$	$[x, y] \mathcal{R}_A [v, z] \iff y = v$	
<i>before</i>	$\langle L \rangle$	$[x, y] \mathcal{R}_L [v, z] \iff y < v$	
<i>started-by</i>	$\langle B \rangle$	$[x, y] \mathcal{R}_B [v, z] \iff x = v \wedge z < y$	
<i>finished-by</i>	$\langle E \rangle$	$[x, y] \mathcal{R}_E [v, z] \iff y = z \wedge x < v$	
<i>contains</i>	$\langle D \rangle$	$[x, y] \mathcal{R}_D [v, z] \iff x < v \wedge z < y$	
<i>overlaps</i>	$\langle O \rangle$	$[x, y] \mathcal{R}_O [v, z] \iff x < v < y < z$	

$$\psi ::= p \mid \neg\psi \mid \psi \vee \psi \mid \langle X \rangle \psi \mid \langle \bar{X} \rangle \psi$$

$$X \in \{A, L, B, E, D, O\}.$$

However, all modalities can be expressed by means of  $\langle A \rangle$ ,  $\langle B \rangle$ ,  $\langle E \rangle$  and their inverse modalities  $\langle \bar{A} \rangle$ ,  $\langle \bar{B} \rangle$ ,  $\langle \bar{E} \rangle$  only

# Kripke structures



An example of Kripke structure

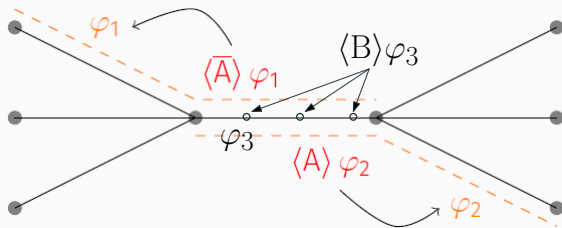
- HS formulas are interpreted over (finite) state-transition systems whose states are labelled with sets of proposition letters (Kripke structures)
- An interval is a **trace** (finite path) in a Kripke structure
- How to determine the properties of a trace (i.e. its proposition letters), in terms of its states?

# HS semantics and model checking

Truth of a formula  $\psi$  over a trace  $\rho$  of a Kripke structure

$\mathcal{K} = (\mathcal{AP}, W, \delta, \mu, w_0)$ :

- $\mathcal{K}, \rho \models p$  iff  $p \in \bigcap_{w \in \text{States}(\rho)} \mu(w)$ , for any letter  $p \in \mathcal{AP}$   
(homogeneity assumption)

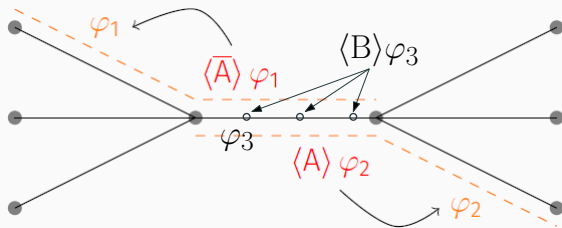


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## Model Checking

$\mathcal{K} \models \psi \iff$  for all *initial* traces  $\rho$  of  $\mathcal{K}$ , it holds that  $\mathcal{K}, \rho \models \psi$

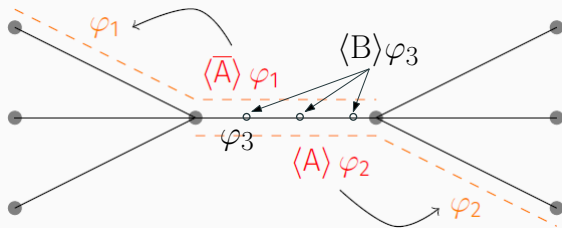
Possibly **infinitely many** traces!

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$\mathcal{K} = (\mathcal{AP}, W, \delta, \mu, w_0)$ :

- $\mathcal{K}, \rho \models \psi$  iff  $p \in \mu(\text{fst}(\rho), \text{lst}(\rho))$ , for any letter  $p \in \mathcal{AP}$  (endpoint-based labeling);



## Model Checking

$\mathcal{K} \models \psi \iff$  for all *initial* traces  $\rho$  of  $\mathcal{K}$ , it holds that  $\mathcal{K}, \rho \models \psi$

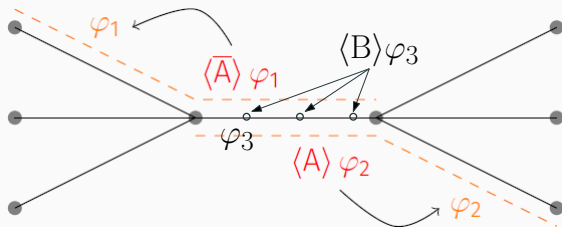
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# HS semantics and model checking

Truth of a formula  $\psi$  over a trace  $\rho$  of a Kripke structure

$\mathcal{K} = (\mathcal{AP}, W, \delta, \mu, w_0)$ :

- $\mathcal{K}, \rho \models \psi$  other semantic variants?



## Model Checking

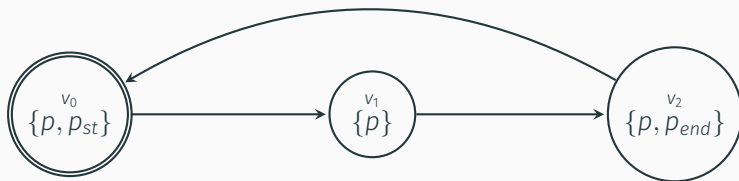
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Possibly **infinitely many** traces!



# Example—Printer

Adapted from [Lomuscio A., Michaliszyn J.]

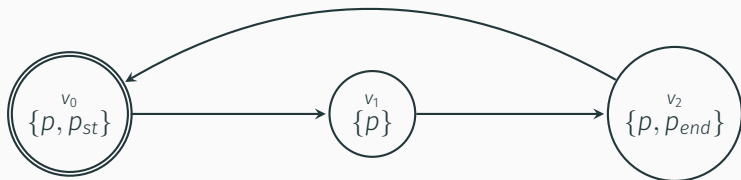


Imagine we want to label the **process of printing** a single sheet of paper with  $p$ .

- Under homogeneity,  
 $v_0v_1v_2$  labeled by  $p \Rightarrow v_0v_1$  and  $v_1v_2$  labeled by  $p$

## Example—Printer

Adapted from [Lomuscio A., Michaliszyn J.]



Imagine we want to label the **process of printing** a single sheet of paper with  $p$ .

- Under endpoint-based labeling, assuming  $p \in \mu(v_0, v_2)$ , then  $(v_0v_1v_2)^n$  are all labeled by  $p$

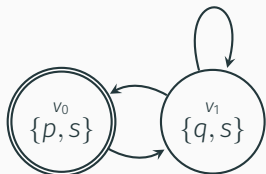
# Our regular expressions

$$r ::= \varepsilon \mid \phi \mid r \cup r \mid r \cdot r \mid r^*$$

where  $\phi$  is a Boolean (propositional) formula over  $\mathcal{AP}$ .

Examples:

- $r_1 = (\mathbf{p} \wedge \mathbf{s}) \cdot \mathbf{s}^* \cdot (\mathbf{p} \wedge \mathbf{s})$
- $r_2 = (\neg \mathbf{p})^*$



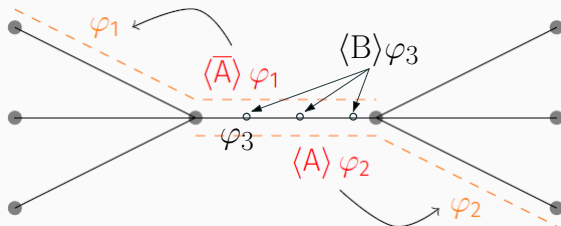
- $\rho = v_0 v_1 v_0 v_1 v_1$
- $\mu(\rho) = \{p, s\}\{q, s\}\{p, s\}\{q, s\}\{q, s\}$
- $\rho' = v_0 v_1 v_1 v_1 v_0$
- $\mu(\rho') = \{p, s\}\{q, s\}\{q, s\}\{q, s\}\{p, s\}$ 
  - $\mu(\rho) \notin \mathcal{L}(r_1)$ , but  $\mu(\rho') \in \mathcal{L}(r_1)$
  - $\mu(\rho) \notin \mathcal{L}(r_2)$ , and  $\mu(\rho) \notin \mathcal{L}(r_2)$

# HS semantics with regular expressions

Truth of a formula  $\psi$  over a trace  $\rho$  of a Kripke structure

$\mathcal{K} = (\mathcal{AP}, W, \delta, \mu, w_0)$ :

- $\mathcal{K}, \rho \models r$  iff  $\mu(\rho) \in \mathcal{L}(r)$ ;



## Model Checking

$\mathcal{K} \models \psi \iff$  for all *initial* traces  $\rho$  of  $\mathcal{K}$ , it holds that  $\mathcal{K}, \rho \models \psi$

Possibly infinitely many traces!

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- $\mathcal{K}, \rho \models r$  iff  $\mu(\rho) \in \mathcal{L}(r)$ ;

- To force **homogeneity**, all regular expressions in the formula:

$$\rho \cdot (p)^*$$

- for **endpoint-based labeling**, regular expressions in the formula:

$$\bigcup_{(i,j) \in I} (q_i \cdot \top^* \cdot q_j)$$

for some  $I \subseteq \{1, \dots, |W|\}^2$ , where  $q_i \in \mathcal{AP}$  labels only  $w_i \in W$ .

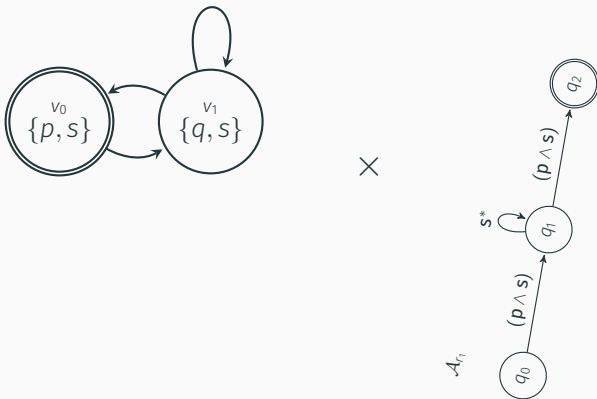
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# Decidability of MC for HS + regular expressions

Given  $\mathcal{K}$  and an HS formula  $\varphi$  over  $\mathcal{AP}$ , we build an NFA over  $\mathcal{K}$  accepting the set of traces  $\rho$  such that  $\mathcal{K}, \rho \models \varphi$ .

Idea: for a regular expression  $r$



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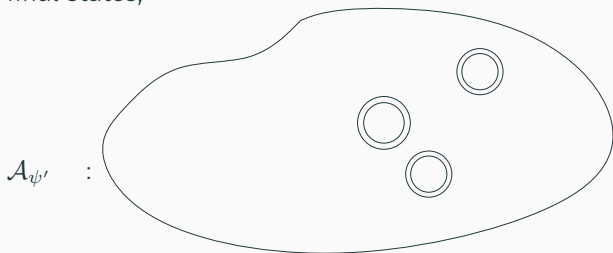
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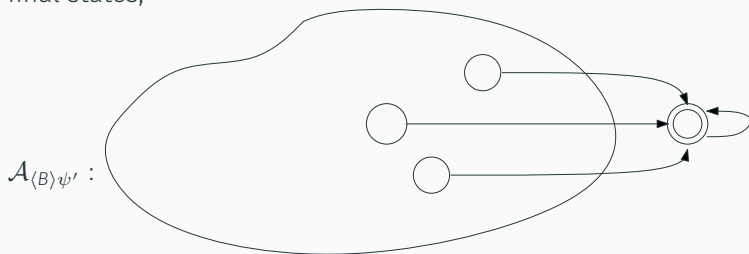
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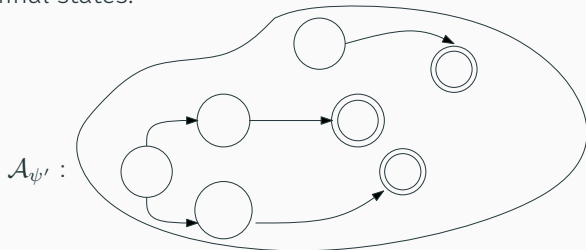
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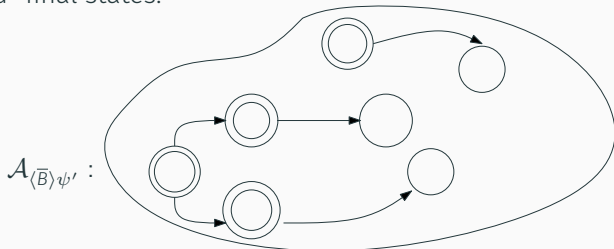
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## Theorem

*The MC problem for HS (+ regular expressions) over finite Kripke structures is decidable.*

# The $\overline{A\overline{A}B\overline{B}}$ fragment + regular expressions

Formulas of  $\overline{A\overline{A}B\overline{B}}$  + regular expressions can be checked by using **polynomial working space**.

The following theorem is a building block of the **PSPACE**-model checking algorithm for  $\overline{A\overline{A}B\overline{B}}$ .

## Theorem (Exponential small-model for $\overline{A\overline{A}B\overline{B}}$ )

Let  $\rho$  be a trace of  $\mathcal{K}$  and  $\varphi$  be an  $\overline{A\overline{A}B\overline{B}}$  formula with RE's  $r_1, \dots, r_u$  such that

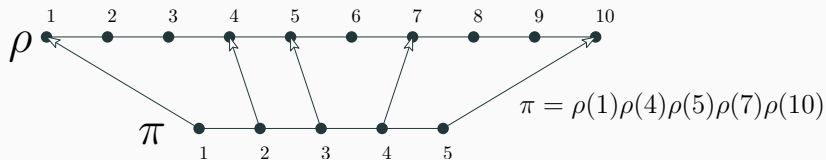
$$\mathcal{K}, \rho \models \varphi.$$

Then, there exists a trace  $\pi$  of  $\mathcal{K}$  such that

$$\mathcal{K}, \pi \models \varphi \quad \text{and} \quad |\pi| \leq |W| \cdot (|\varphi| + 1) \cdot 2^{2 \sum_{\ell=1}^u |r_{\ell}|}.$$

Proved by a **contraction technique**.

# Small model for $A\bar{A}B\bar{B}$

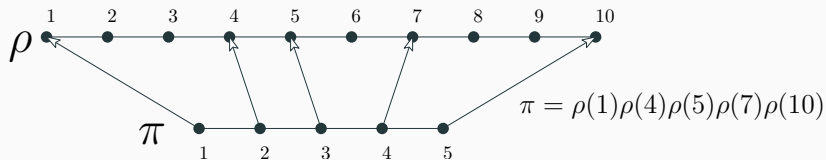


We guarantee that:

for all corresponding positions  $i, j$ :  $\mathcal{A}^s(\mu(\pi[1 \dots j])) = \mathcal{A}^s(\mu(\rho[1 \dots i]))$



# Small model for $\overline{AAB\overline{B}}$



We guarantee that:

for all corresponding positions  $i, j$ :  $\mathcal{A}^s(\mu(\pi[1 \dots j])) = \mathcal{A}^s(\mu(\rho[1 \dots i]))$

If  $\mathcal{K}, \rho \models \varphi \Rightarrow$  there is a trace  $\pi$  of  $\mathcal{K}$ , induced by  $\rho$ , such that

$$\mathcal{K}, \pi \models \varphi \quad \text{and} \quad |\pi| \leq |W| \cdot (|\varphi| + 1) \cdot 2^{2 \sum_{\ell=1}^u |r_\ell|}.$$

This small model is “strict”!

# A PSPACE MC algorithm for $\overline{A\overline{A}B\overline{B}}$ —Trials

- The algorithm can consider only traces having length bounded by the exponential small model
- However, they are still **too long!**  $\Rightarrow$  **triples summarizing traces,**

$$(G, D(\psi), w)$$

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•  $G \subseteq \text{Subf}_{(B)}(\psi)$

contains the **subformulas that hold on some prefix**

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- $D(\psi)$  is the **configuration of the DFAs** after reading the trace (only the current states, not the whole DFAs!),

# A PSPACE MC algorithm for $\overline{A\overline{A}B\overline{B}}$ —Trials

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- $w$  is the last state of the trace



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- However, they are still **too long!**  $\Rightarrow$  **triples summarizing traces,**

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## Lemma

If  $\rho$  and  $\rho'$  summarized by the same triple  $(G, D(\psi), w)$ , then  $(\psi \in \overline{B\overline{B}})$

$$\mathcal{K}, \rho \models \psi \iff \mathcal{K}, \rho' \models \psi.$$

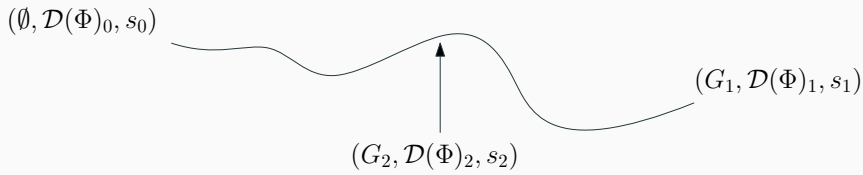
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$(\emptyset, \mathcal{D}(\Phi)_0, s_0)$



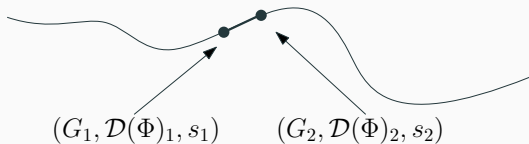
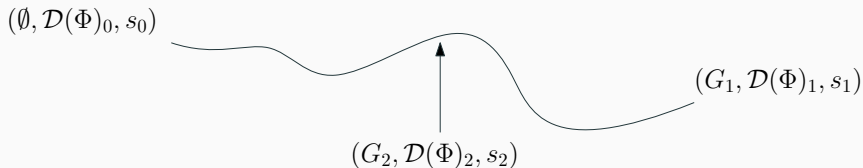
$(G_1, \mathcal{D}(\Phi)_1, s_1)$

# A PSPACE MC algorithm for $A\bar{A}B\bar{B}$

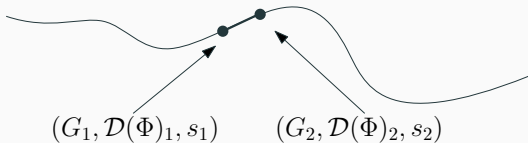
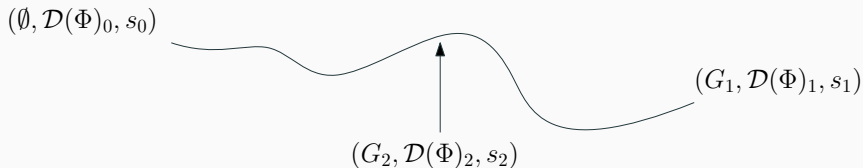




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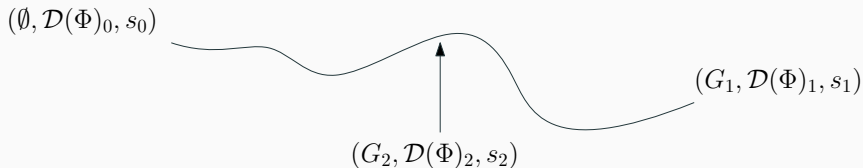


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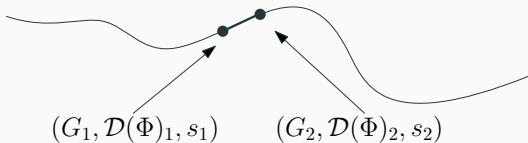


Compatible?

# A PSPACE MC algorithm for $\overline{A\overline{A}B\overline{B}}$



Bisection  $\rightsquigarrow$  polynomial depth recursion



Compatible?

# A PSPACE MC algorithm for $\overline{A\overline{A}B\overline{B}}$

---

## Algorithm 1 Check( $\mathcal{X}, \psi, w, G, D(\psi)$ )

---

- 1: **if**  $\psi = r$  **then**  *$\triangleleft r$  is a regular expression*
  - 2:     **if** the current state of the DFA for  $r$  in  $\text{advance}(D(\psi), \mu(w))$  is final **then**
  - 3:         **return**  $\top$
  - 4:     **else**
  - 5:         **return**  $\perp$
  - 6: **else if**  $\psi = \neg\psi'$  or  $\psi = \psi_1 \wedge \psi_2$  **then**
  - 7:     Call Check recursively
  - 8: **else if**  $\psi = \langle B \rangle \psi'$  **then**
  - 9:     **return**  $\psi' \in G$
  - 10: **else if**  $\psi = \langle \overline{B} \rangle \psi'$  **then**
  - 11:     **for** all  $b \in \{1, \dots, |W| \cdot (2^{|\psi'|} + 1) \cdot 2^{2 \sum_{e=1}^u |r_{e1}|} - 1\}$ , all  $(G', D(\psi)', w')$  **do**
  - 12:         **if**  $\text{Reach}(\mathcal{X}, \psi', (G, D(\psi), w), (G', D(\psi)', w'), b)$  and  $\text{Check}(\mathcal{X}, \psi', w', G', D(\psi)')$  **then**
  - 13:             **return**  $\top$
  - 14:     **return**  $\perp$
-

# A PSPACE MC algorithm for $A\bar{A}\bar{B}\bar{B}$

---

## Algorithm 2 Check( $\mathcal{X}, \psi, w, G, D(\psi)$ )

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- 1: **if**  $\psi = r$  **then**  *$\triangleleft r$  is a regular expression*
  - 2:     **if** the current state of the DFA for  $r$  in **advance**( $D(\psi), \mu(w)$ ) is final **then**
  - 3:         **return**  $\top$
  - 4:     **else**
  - 5:         **return**  $\perp$
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  - 10: **else if**  $\psi = \langle \bar{B} \rangle \psi'$  **then**
  - 11:     **for all**  $b \in \{1, \dots, |W| \cdot (2^{|\psi'|} + 1) \cdot 2^{2 \sum_{e=1}^u |r_e|} - 1\}$ , all  $(G', D(\psi)', w')$  **do**
  - 12:         **if** **Reach**( $\mathcal{X}, \psi', (G, D(\psi), w), (G', D(\psi)', w'), b$ ) and **Check**( $\mathcal{X}, \psi', w', G', D(\psi)'$ ) **then**
  - 13:             **return**  $\top$
  - 14:     **return**  $\perp$
-

## Theorem

The MC problem for formulas of  $\overline{A\overline{A}B\overline{B}}$  (+ regular expressions) over finite Kripke structures is **PSPACE**-complete.

## Proof.

The purely propositional fragment of HS is hard for **PSPACE**:  
reduction from the **PSPACE**-complete *universality problem for regular expressions*. □

# Complexity results

	Homogeneity	Regular expressions
Full HS, BE	non-elementary EXPSpace-hard	non-elementary EXPSpace-hard
$A\bar{A}B\bar{B}\bar{E}, A\bar{A}E\bar{B}\bar{E}$	EXPSpace PSPACE-hard	non-elementary PSPACE-hard
$A\bar{A}\bar{B}\bar{E}$	PSPACE-complete	non-elementary PSPACE-hard
$A\bar{A}B\bar{B}, B\bar{B}, \bar{B},$ $A\bar{A}E\bar{E}, E\bar{E}, \bar{E}$	PSPACE-complete	PSPACE-complete
$A\bar{A}B, A\bar{A}E, AB, \bar{A}E$	$P^{NP}$ -complete	PSPACE-complete
$A\bar{A}, \bar{A}B, AE, A, \bar{A}$	$P^{NP[O(\log^2 n)]}$ $P^{NP[O(\log n)]}$ -hard	PSPACE-complete
Prop, B, E	co-NP-complete	PSPACE-complete

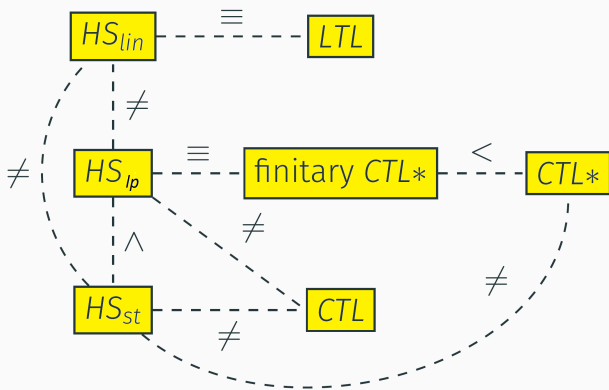
# Complexity results




	Homogeneity	Regular expressions
Full HS, BE	non-elementary EXPSPACE-hard	non-elementary EXPSPACE-hard
$A\bar{A}B\bar{B}\bar{E}, A\bar{A}E\bar{B}\bar{E}$	EXPSPACE PSPACE-hard	non-elementary PSPACE-hard
$A\bar{A}\bar{B}\bar{E}$	PSPACE-complete	non-elementary PSPACE-hard
$A\bar{A}B\bar{B}, B\bar{B}, \bar{B},$ $A\bar{A}E\bar{E}, E\bar{E}, \bar{E}$	PSPACE-complete	PSPACE-complete
$A\bar{A}B, A\bar{A}E, AB, \bar{A}E$	$P^{NP}$ -complete	PSPACE-complete
$A\bar{A}, \bar{A}B, AE, A, \bar{A}$	$P^{NP[O(\log^2 n)]}$ $P^{NP[O(\log n)]}$ -hard	PSPACE-complete
Prop, B, E	co-NP-complete	PSPACE-complete



- Determining the precise **complexity of full HS**
- MC over **visibly pushdown systems** (for recursive programs, infinite state systems)
- Application: **Planning as Model Checking** with HS
- Come up with inherently **interval-based models** of systems (Kripke structures are oriented to the description of point-based properties of systems, and their state-by-state evolution)

# Expressiveness comparison



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